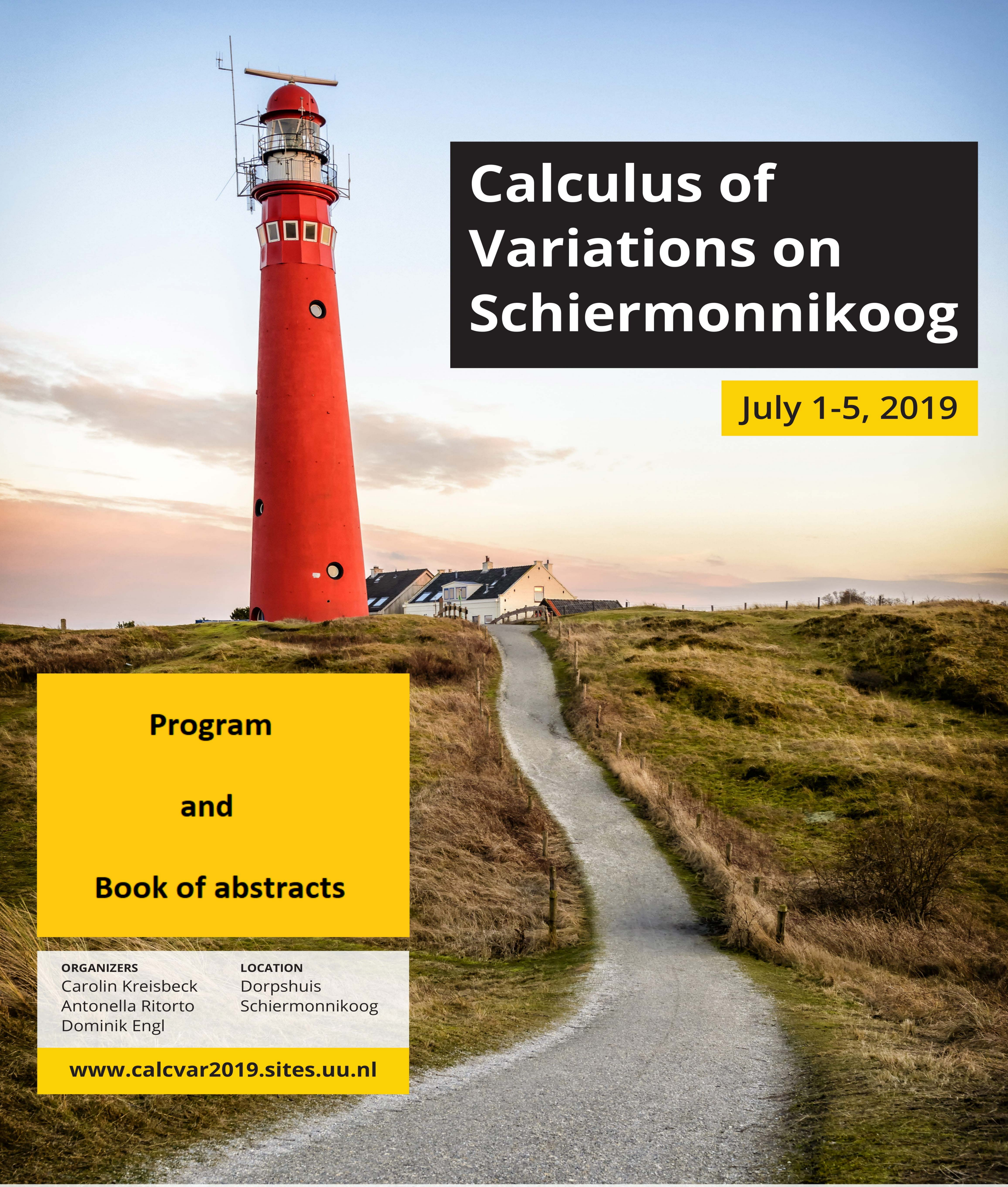

Calculus of Variations on Schiermonnikoog

July 1-5, 2019

Program

and

Book of abstracts

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NWO
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NDNS+

Program

Monday		
8:30-9:00	Registration	
9:00:-9:15	Opening	
9:15-9:55	Irene Fonseca	Second order structured deformations in the space of functions of bounded Hessian
9:55-10:35	Elvira Zappale	Optimal Design Problems
10:35-11:00	Coffee	
11:00-11:40	Manuel Friedrich	Compactness and Gamma-convergence of free discontinuity problems
11:40-12:20	Elisa Davoli	Adaptive image processing: first order PDE constraint regularizers and bilevel training scheme
12:20-14:20	Lunch	
14:20-15:00	Matthias Röger	Intersecting phase boundaries in phase field approximations of the Willmore functional
15:00-15:40	Patrick Dondl	A phase-field approximation of the perimeter under a connectedness constraint
15:40-16:10	Coffee	
16:10-16:35	Augusto Gerolin	Entropy-Transport: Duality and convergence of the IPFP
16:35-17:00	Martin Jesenko	Existence of a blocking line in the quenched Edwards-Wilkinson model

Tuesday		
9:00-9:40	Georg Dolzmann	Evolution of vector fields on flexible curves and surfaces
9:40-10:20	Dorothee Knees	Optimal control of rate-independent systems
10:20-10:50	Coffee	
10:50-11:15	Paolo Gidoni	An introduction to rate-independent soft crawlers
11:15-11:40	Artur Stephan	EDP-Convergence for linear Reaction-Diffusion Systems with different time scales
11:40-12:20	Oliver Tse	Dynamical-variational transport costs: Towards a framework for "generalised" gradient flows
12:20-14:20	Lunch	
14:20-15:00	Giovanni Leoni	A variational approach for water waves
15:00-15:25	Valerio Pagliari	Approximation of the mean curvature flow by nonlocal curvature motions
15:25-16:20	Poster + Coffee	
16:20-17:00	Marco Morandotti	Spatially inhomogeneous evolutionary games

Wednesday		
9:00-9:40	Andrea Braides	Limits of convolution functionals
9:40-10:20	Marco Cicalese	Chirality transitions in magnetic materials
10:20-10:50	Coffee	
10:50-11:15	Annika Bach	Finite-Difference Discretizations of the Ambrosio-Tortorelli Functionals on periodic and on random lattices
11:15-11:40	Janusz Ginster	Upscaling in Molecular Solvation
11:40-12:20	Filippo Cagnetti	Stochastic Homogenisation of Free-Discontinuity Problems
12:20-14:20	Lunch	
14:20-18:00	Social activity	
18:00	Conference dinner	

Thursday		
9:30-10:10	Carlos Mora-Corral	Regularity and invertibility for orientation-preserving Sobolev maps
10:10-10:50	Stefan Krömer	Injective nonlinear elasticity via penalty terms: analysis and numerics
10:50-11:15	Coffee	
11:15-11:55	Giuliano Lazzaroni	Derivation of linear elasticity from multiwell energies
11:55-12:20	Stephan Wojtowytsch	Elastic curves and the buckling of cylindrical shells
12:20-14:20	Lunch	
14:20-15:00	Rita Ferreira	Lower-dimensional models for multi-domains involving bending-torsion moments
15:00-15:40	Igor Velčić	Derivation of homogenized Euler-Lagrange equations for von Karman rod
15:40-16:10	Coffee	

Friday		
9:00-9:40	Martin Kružík	Derivation of von Kármán plate theory in the framework of three-dimensional viscoelasticity
9:40-10:05	Antonella Ritorto	Critical elliptic problems for α-stable processes
10:05-10:30	Cornelia Mihaila	Axial Symmetry for Fractional Capillarity Droplets
10:30-11:00	Coffee	
11:00-11:40	John Ball	Remarks on the Oseen-Frank theory of liquid crystals
11:40-11:45	Closing	
11:45	Lunch	

Book of abstracts

Invited talks

John Ball (Heriot-Watt University)

Remarks on the Oseen-Frank theory of liquid crystals

The lecture will describe some recent results on the Oseen-Frank theory of nematic liquid crystals, with Epifanio Virga on energy-minimizing properties of universal solutions, and with Lu Liu on 2D exterior problems.

Andrea Braides (University of Rome Tor Vergata)

Limits of convolution functionals

Starting from the model case of convolution functionals approximating the Dirichlet integral, we provide a general theory of limits of convolution energies with a strong connection with discrete-to-continuum results obtained in the last twenty years. We apply such a theory to stochastic homogenization, perforated domains and thin-film limits. Works in collaboration with R. Alicandro, N. Ansini, A. Piatnitski and A. Tribuzio.

Filippo Cagnetti (University of Sussex)

Stochastic Homogenisation of Free-Discontinuity Problems

We will discuss the stochastic homogenisation of free-discontinuity functionals. Assuming stationarity for the random volume and surface integrands, we prove the existence of a homogenised functional, whose volume and surface integrands are characterised by asymptotic formulas involving minimisation problems on larger and larger cubes with special boundary conditions. In the proof we combine a recent deterministic Gamma-convergence result for free-discontinuity functionals with the Subadditive Ergodic Theorem by Akcoglu and Krengel. This is a joint work in collaboration with Gianni Dal Maso (SISSA), Lucia Scardia (Heriot-Watt University), and Caterina Zeppieri (University of Münster).

Marco Cicalese (Technische Universität München)

Chirality transitions in magnetic materials

We study a ferromagnetic/antiferromagnetic lattice model and prove that the excess energy about the ground state can be rescaled to highlight the emergence of chirality transitions. The result is obtained by computing the Gamma-limit of the discrete system as the lattice spacing vanishes.

Elisa Davoli (University of Vienna)

Adaptive image processing: first order PDE constraint regularizers and a bilevel training scheme

In this talk we will introduce a novel class of image-processing regularizers, providing a unified approach to standard regularizers such as total variation and total generalized variation functionals. By means of a bilevel training scheme we will identify optimal parameters and regularizers. Finally, by Gamma-convergence we will show existence of a solution for any given set of training imaging data. This is joint work with Irene Fonseca and Pan Liu.

Georg Dolzmann (Universität Regensburg)

Evolution of vector fields on flexible curves and surfaces

In this lecture we discuss some recent progress on a model system consisting of a flexible surface and a vector field defined on the surface in the case in which an interaction between the vector field and the conformation of the surface is present. Recent approaches towards the existence of solutions will be reviewed and short time existence will be established. Under additional assumptions some long time existence results can be stated.

This is joint work with Christopher Brand (Regensburg) and Alessandra Pluda (Pisa).

Patrick Dondl (University of Freiburg)

A phase-field approximation of the perimeter under a connectedness constraint

We develop a phase-field approximation of the relaxation of the perimeter functional in the plane under a connectedness constraint based on the classical Modica-Mortola functional and a diffuse quantitative version of path-connectedness. We prove convergence of the approximating energies and present numerical results and applications to image segmentation. Join work with M. Novaga (Pisa), B. Wirth (Münster), and S. Wojtowytsch (Pittsburgh).

Rita Ferreira (KAUST)

Lower-dimensional models for multi-domains involving bending-torsion moments

In this talk, we address a dimension-reduction problem in the context of nonlinear elasticity where the applied external surface forces induce bending-torsion moments. The underlying body is a multi-structure in $\mathbb{R}^3$ consisting of a thin tube-shaped domain placed upon a thin plate-shaped domain. The problem involves two small parameters, the radius of the cross-section of the tube-shaped domain and the thickness of the plate-shaped domain. We characterize the different limit models, including the limit junction condition, in the membrane-string regime according to the ratio between these two parameters as they converge to zero.

This is a joint work with Elvira Zappale (University of Salerno, Italy).

Irene Fonseca (Carnegie Mellon University)

Second order structured deformations in the space of functions of bounded Hessian

An integral representation is obtained in BH in the vein of the global method for relaxation, and is applied to a relaxation problem in the context of structured deformations.

Manuel Friedrich (WWU Münster)

Compactness and Gamma-convergence for free discontinuity problems

In this talk, I present a general compactness result in the space GSBV which extends the classical statement due to Ambrosio to problems without a priori bounds on the functions. As an application, we revisit Gamma-convergence results for free discontinuity functionals comprising bulk and surface energies: we investigate sequences of boundary value problems and show convergence of minimum values and minimizers. Afterwards, I will present analogous results for problems involving only surface energies defined on piecewise rigid functions, i.e., functions which are piecewise affine on a Caccioppoli partition whose derivative in each component is constant and lies in a set without rank-one connections.

Dorothee Knees (University of Kassel)

Optimal control of rate-independent systems

Rate independent systems can be formulated based on an energy functional and a dissipation potential that is assumed to be convex, lower semicontinuous and positively homogeneous of degree one. Here, we will focus on the nonconvex case meaning that the energy is not convex. In this case, the solution typically is discontinuous in time. There exist several (in general not equivalent) notions of weak solutions. We focus on so-called balanced viscosity solutions, discuss the properties of solution sets and discuss the well posedness of an optimal control problem for such systems.

Stefan Krömer (UTIA, Czech Academy of Sciences)

Injective nonlinear elasticity via penalty terms: analysis and numerics

I will present some new ideas for nonlinear elasticity with a global injectivity constraint preventing self-interpenetration of the elastic body. Our main focus are penalization terms replacing the injectivity constraint (the Ciarlet-Necas condition). Among other things, the penalization can be chosen in such a way that self-interpenetration is prevented even at finite value of the penalization parameter, and not just in the limit. Our penalty method provides a working numerical scheme with provable convergence along a subsequence, for models of non-simple materials (including a term with higher order derivatives).

This is joint work with Jan Valdman (UTIA AVCR, Prague).

Martin Kružík (Czech Academy of Sciences)

Derivation of von Kármán plate theory in the framework of three-dimensional viscoelasticity

We apply a quasistatic nonlinear model for nonsimple viscoelastic materials at a finite-strain setting in the Kelvin's-Voigt's rheology to derive a viscoelastic plate model of von Kármán type. We start from solutions to a model of three-dimensional viscoelasticity for 2nd-grade materials where the viscosity stress tensor complies with the principle of time-continuous frame-indifference.

Combining the derivation of nonlinear plate theory by Friesecke, James and Müller, and the abstract theory of gradient flows in metric spaces by Sandier and Serfaty we perform a dimension-reduction from 3D to 2D and identify weak solutions of viscoelastic form of von Kármán plates. This is a joint work with M. Friedrich (Münster).

Giuliano Lazzaroni (University of Florence)

Derivation of linear elasticity from multiwell energies

Linear elasticity can be rigorously derived from finite elasticity under the assumption of small loadings in terms of Gamma-convergence. This was first done in the case of one-well energies with super-quadratic growth and later generalised to different settings, in particular to the case of multi-well energies where the distance between the wells is very small (comparable to the size of the load). In this talk we discuss the case when the distance between the wells is independent of the size of the load. In this context linear elasticity can be derived by adding to the multi-well energy a singular higher order term which penalises jumps from one well to another. The size of the singular term has to satisfy certain scaling assumptions whose optimality is shown in most of the cases. Finally, the derivation of linear elasticity from a discrete model is provided, showing that the role of the singular perturbation term is played in this setting by interactions beyond nearest neighbours.

Joint work with R. Alicandro, G. Dal Maso, M. Palombaro.

Giovanni Leoni (Carnegie Mellon University)

A variational approach for water waves

We study the existence and regularity of non-trivial solutions to a one-phase Bernoulli free boundary problem with mixed periodic-Dirichlet boundary conditions. This is joint work with Giovanni Gravina.

Carlos Mora-Corral (Universidad Autónoma de Madrid)

Regularity and invertibility for orientation-preserving Sobolev maps

The regularity of $W^{1,p}$ Sobolev maps differs drastically according to whether the exponent p of integrability is $p > n$ or $p < n$, where n is the dimension of the space. In this talk, we define a class of Sobolev deformations that are orientation-preserving and suitable for models in nonlinear elasticity. The main feature of these maps is that they do not create cavities. The exponent p satisfies $p > n-1$ and its regularity properties (continuity, differentiability, monotonicity, Lusin's condition) are similar to their counterparts with $p > n$. We also show local and global invertibility theorems for this class. We give applications to existence results in variational models in Mechanics whose energy involves both the reference and deformed configurations, like liquid crystal elastomers and magnetoelasticity.

Joint work with M. Barchiesi, D. Henao and M. Oliva.

Marco Morandotti (Politecnico di Torino)

Spatially inhomogeneous evolutionary games

We study an interaction model of a large population of players based on an evolutionary game, which describes the dynamical process of how the distribution of strategies changes in time according to their individual success.

Differently from spatially homogeneous dynamical games, we assume that the population of players is distributed over a state space and that they are each endowed with probability distributions of pure strategies, which they draw at random to evolve their states. Simultaneously, the mixed strategies evolve according to a replicator dynamics, modeling the success of pure strategies according to a payoff functional.

We establish existence, uniqueness, and stability of Lagrangian and Eulerian solutions of this dynamical game by using methods of ODE and optimal transport on Banach spaces.

Matthias Röger (Technische Universität Dortmund)

Intersecting phase boundaries in phase field approximations of the Willmore functional

We consider a well-known phase field approximation of the Willmore energy, which was in its original form introduced by De Giorgi. The approximation has been rigorously justified for smooth phase boundaries and small space dimensions. We investigate the occurrence of intersecting phase boundaries and discuss a new approach that avoids such non-generic configurations. (This is joint work with C. Zwilling and A. Rätz, TU Dortmund)

Oliver Tse (TU Eindhoven)

Dynamical-variational transport costs: Towards a framework for "generalised" gradient flows.

In this talk, dynamical-variational transport costs (DVTs) will be introduced and motivated. The role in which these objects play in the theory of (generalised) gradient flows will be illustrated with an example on Markov jump processes. Finally, open questions and challenges will be mentioned.

Igor Velčić (University of Zagreb)

Derivation of homogenized Euler-Lagrange equations for von Karman rod

We study the effects of simultaneous homogenization and dimension reduction in the context of convergence of stationary points for thin nonhomogeneous rods under the assumption of the von Karman scaling. Assuming stationarity condition for a sequence of deformations close to a rigid body motion, we prove that the corresponding sequences of scaled displacements and twist functions converge to a limit point, which is the stationary point of the homogenized von Karman rod model. The analogous result holds true for the von Karman plate model and bending rod model. This is a joint work with M. Bukal (University of Zagreb) and M. Pawelczyk (TU Dresden).

Elvira Zappale (Università di Salerno)

Optimal Design Problems

Results devoted to obtain a measure representation for functionals arising in the context of optimal design problems will be presented. Aiming at the description of several applications, different sets of assumptions will be considered.

Contributed talks

Annika Bach (TU München)

Finite-Difference Discretizations of the Ambrosio-Tortorelli Functionals on periodic and on random lattices

Motivated by applications to image reconstruction, in this talk we present recent results on the asymptotic behavior finite-differences discretizations of the Ambrosio-Tortorelli functionals, the latter providing an elliptic approximation of the Mumford-Shah functional.

In a first part we consider a discretization on a *periodic* lattice with lattice spacing δ and we show that the discretized functionals Γ-converge to the Mumford-Shah functional only if $\delta \ll \varepsilon$, ε being the approximation parameter of the Ambrosio-Tortorelli functionals. If instead δ and ε are of the same order the structure of the underlying periodic lattice affects the Γ-limit, which turns out to be an anisotropic version of the Mumford-Shah functional.

In a second part we show that the emergence of anisotropy in the Γ-limit in the regime $\delta \sim \varepsilon$ can be avoided when discretizing on a stationary, ergodic and isotropic *random* lattice. In fact, we will see that a finite-difference discretization on such a lattice Γ-converges to the Mumford-Shah functional when the lattice parameter δ is proportional to the elliptic parameter ε.

The first part of the talk is based on a joint work with Andrea Braides (Rome) and Caterina Ida Zeppieri (Münster), while the second part is based on a joint work with Marco Cicalese (Munich) and Matthias Ruf (Brussels).

Augusto Gerolin (Vrije Universiteit Amsterdam)

Entropy-Transport: Duality and convergence of the IPFP

We want to exploit the equivalence between the Schrödinger problem and the entropy penalized optimal transport problem, in order to find a different approach to the duality, much more in the spirit of optimal transport. This approach will result in a priori estimates which are consistent in the limit when epsilon goes to zero. Our method extends easily also when we have more that two marginals: as a byproduct we prove that the IPFP algorithm is converging also in the multimarginal case.

Paolo Gidoni (Czech Academy of Sciences)

An introduction to rate-independent soft crawlers

The inclusion of elastic components in the modelling and design of biomimetic crawlers endows these systems with new compliance capabilities, but at the same time raises additional challenges to the analysis of their locomotion properties. The mathematical theory of rate-independent systems and sweeping processes provides an effective framework to address such issues. Indeed, the various strategies adopted by crawlers to achieve locomotion, such as friction anisotropy, complex shape changes and control on the friction coefficients, can be effectively described in terms of stasis domains. The aim of this talk is to provide, with the aid of representative toy models, an essential introduction to the modelling of rate-independent soft crawlers; we highlight the differences between a mechanical systems guided by active external forces and and soft self-propelled locomotors, and discuss the new issues raised by the latter.

Janusz Ginster (TU Berlin)

Upscaling in Molecular Solvation

We treat the variational Poisson-Boltzmann model of molecular solvation of discrete polar macromolecules in a nonpolar continuous solvent. We show that as the size of the macromolecules tends to zero and their number to infinity, there are different scaling regimes where either the self-energies of optimal molecule clusters or long-range Coulombic interactions dominate, depending on the relative density. Further, the presence of common ions in the solvent leads to a screening effect between different molecule clusters, negating long-range interactions.

Martin Jesenko (Albert-Ludwigs-Universität Freiburg)

Existence of a blocking line in the quenched Edwards-Wilkinson model

The equation

$$\partial_t u(t,x) = \Delta u(t,x) - f(x, u(t,x)) + F$$

arises in several models in physics, e.g., when exploring charge density waves, interfaces or contact lines (see [1, 2]). Here is f a local driving force and F an external force. Since f is assumed to be random but time-independent, this model is frequently called the quenched Edwards-Wilkinson model.

We suppose f to come from "obstacles", i.e. from small portions of the medium with different properties, and may be positive or negative. For one-dimensional case we show that there is a critical radius of these obstacles such that there exists a blocking line in spite of the presence of negative obstacles.

[1] S. Brazovskii, T. Nattermann, Adv. in Phys. 53, 2 (2004), p. 177–252.

[2] M. Kardar, Phys. Rep. 301, 1-3 (1998), p. 85–112.

Cornelia Mihaila (University of Chicago)

Axial Symmetry for Fractional Capillarity Droplets

A classical result of Wente, motivated by the study of sessile capillarity droplets, shows the axial symmetry of every hypersurface which meets a hyperplane at a constant angle and has mean curvature depending only on the distance from that hyperplane. We will prove an analogous result for the fractional mean curvature operator.

Valerio Pagliari (Università di Pisa)

Approximation of the mean curvature flow by nonlocal curvature motions

We consider the L^2-gradient flow of a nonlocal perimeter functional, or, in other words, a motion by nonlocal curvature. It is known that the well-posedness of the Cauchy problem for the flow can be established via a level-set formulation within the framework of viscosity solutions; roughly speaking, we are interested in their large scale limit. More precisely, we show that a suitable rescaling of the nonlocal curvature induces a localisation effect, that is, as the scaling parameter tends to 0, we retrieve an anisotropic mean curvature functional. From this fact, using the theory of geometric barriers introduced by De Giorgi, we prove that the solutions to the rescaled nonlocal motions locally uniformly converge to the solution of the local flow. This is a joint work with A. Cesaroni (Padova).

Antonella Ritorto (Universiteit Utrecht)

Critical elliptic problems for α–stable processes

In this talk, we show the existence of an extremal function for a Sobolev inequality for α-stable operators. To this aim, we obtain a concentration-compactness principle for α–stable processes in $\mathbb{R}^N$. As an application, we obtain the existence of positive solutions for perturbations of critical problems.

Joint work (in progress) with Fernando Quirós (Universidad Autónoma de Madrid), and Arturo de Pablo (Universidad Carlos III, Madrid).

Artur Stephan (WIAS-Berlin)

EDP-Convergence for linear Reaction-Diffusion Systems with different time scales

We study a linear reaction-diffusion system involving slow and fast reactions and investigate its behaviour if some reaction rates tend to infinity. Assuming detailed balance, the problem can be understood as a gradient flow in Wasserstein space. We show how an effective limiting system can be rigorously derived in such a way that the underlying gradient structure is preserved. The limiting process is a reaction-diffusion system with mixed diffusion coefficients coarse-grained with respect to the microscopic equilibria of the fast reactions.

Stephan Wojtowytsch (Carnegie Mellon University)

Elastic curves and the buckling of cylindrical shells

We will discuss a geometric problem for elastic curves in the unit disc motivated by the buckling of thin cylindrical shells. The example is purely geometric and asymptotically reduces to an obstacle problem on the real line which can be solved explicitly using classical techniques. This allows a first order energy expansion with a sharp constant depending only on material parameters.

Posters

Omar Boussaid (Hassiba Benbouali University of Chlef)

Characterizations of symmetric polyconvexity

Symmetric quasiconvexity plays a key role for energy minimization in geometrically linear elasticity theory. Due to the complexity of this notion, a common approach is to retreat to necessary and sufficient conditions that are easier to handle. This poster focuses on symmetric polyconvexity, which is a sufficient condition. We prove a new characterization of symmetric polyconvex functions in the two- and three-dimensional setting, and use it to investigate relevant subclasses like symmetric polyaffine functions and symmetric polyconvex quadratic forms. In particular, we provide an example of a symmetric rank-one convex quadratic form in 3d that is not symmetric polyconvex. The construction takes the famous work by Serre from 1983 on the classical situation without symmetry as inspiration. Beyond their theoretical interest, these findings may turn out useful for computational relaxation and homogenization.

This is a joint work with Pr. Anja Schlömerkemper from University of Wuerzburg and Dr. Carolin Kreisbeck from Utrecht University.

Marin Bužančić (University of Zagreb)

Spectral analysis of thin domains in high-contrast regime

In this work, we consider the system of equations describing three-dimensional thin plates in linearized elasticity with high contrast in the coefficients. We analyse the resolvent problem and the approximating properties of the spectrum when the period of oscillation of the heterogeneous material ε and the thickness of the thin body h simultaneously tend to zero. In order to derive the limit models, we use two-scale convergence results adapted for dimension reduction. By dividing the problem into two invariant subspaces, we are able to prove different behaviours of eigenvalues for bending and membrane displacements.

This is a joint work with Igor Velčić and Josip Žubrinić (Faculty of Electrical Engineering and Computing, University of Zagreb).

Dominik Engl (Universiteit Utrecht)

An analytical discussion of polycrystalline structures in single-slip crystal plasticity

Many solids, such as metals, are polycrystals, meaning that they consist of a cluster of differently rotated single crystals. Such grain structures are known to impose restrictions on still finer microstructures, and highly influence the effective material response. We consider a model in the context of finite crystal plasticity with one active slip system and linear hardening under the assumption of elastically rigid behavior. Mathematically, the possible deformations of the material can be described by an inhomogeneous nonlinear differential inclusion subject to suitable boundary conditions.

In this poster, we discuss different questions related to the solvability of these inclusions and their relaxed versions. The answers depend highly on the interplay between the orientation of the slip systems and the grain geometry. We present a geometry-independent sufficient condition by characterising globally affine solutions. Applying a generalized Hadamard compatibility condition to the boundary grains leads to a necessary condition, which turns out to be also sufficient for certain geometries. By explicit construction, we provide examples of piecewise affine solutions to the relaxed differential inclusion, allowing us to obtain Lipschitz solutions to the unrelaxed problem via convex integration theory. This is joint work with Carolin Kreisbeck (Utrecht University).

Alberto Maione (University of Trento)

Γ-convergence for integral functionals depending on vector fields

Given $\Omega \subset \mathbb{R}^n$ open and bounded, $X(x) = (X_1(x), .., X_m(x))$, $x \in \Omega$ a family of Lipschitz vector fields $X_i(x) = \sum_{j=1}^n c_{ij}(x)\partial_j$ on Ω, $i = 1, .., m$, with $m \le n$ and given $f_h : \Omega \times \mathbb{R}^m \to [0, \infty)$, we discuss Γ-convergence results for the following integral functionals $F_h : L^p(\Omega) \to [0, \infty]$ $(h = 1, 2, ..)$

$$F_h(u) := \begin{cases} \int_\Omega f_h(x, Xu(x))\,dx & \text{if } u \in C^1(\Omega) \\ \infty & \text{in } L^p(\Omega) \setminus C^1(\Omega) \end{cases}$$

under suitable assumptions on the family X and the sequence $(f_h)_h$.

This is a joint work with A. Pinamonti (Trento) and F. Serra Cassano (Trento).

[1] A. Maione, A. Pinamonti, F. Serra Cassano, *Γ-convergence for functionals depending on vector fields. I. Integral representation and compactness*, Preprint available on arXiv 1904.06454 (2019).

Valerio Pagliari (Università di Pisa)

A nonlocal notion of calibration

We focus on Plateau's problem for nonlocal perimeters, that is, we study their minimisation under prescribed boundary conditions, Nonlocal perimeters are a class of geometric functionals, which, qualitatively, express weighted interactions between a given set in the Euclidean space and its complement.

The weight is encoded by a positive, possibly singular kernel. Once existence of solutions to Plateau's problem is established, we introduce a suitable notion of calibration, and, as in the classical setting, we show that the existence of a calibration is a sufficient condition for optimality.

By virtue of this criterion, we can prove that halfspaces are local minimisers for nonlocal perimeters.